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Engineering Notes

Solar Sail Capabilities to Reach Elliptic Rectilinear Orbits

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Nomenclature

a = semimajor axis, AU
a_c = spacecraft characteristic acceleration, mm/s^2
h = angular momentum modulus, $AU^2/year$
r = radial distance, AU
t = time, years
u = radial velocity component, km/s
v = transverse velocity component, km/s
ΔV = velocity variation, km/s
θ = polar angle, deg
$\mu_\odot$ = sun's gravitational parameter, km^3/s^2

Subscripts

a = apocenter
min = minimum
0 = initial
1 = probe's release (end of sailing mode)
2 = probe's destruction (end of scientific mode)
$\oplus$ = Earth

Introduction

AN ELLIPTIC rectilinear orbit (ERO) is a particular conic orbit in which the semimajor axis takes a finite value and the eccentricity is unity [1]. From a geometrical point of view an ERO is a line segment connecting both orbital foci: one endpoint of the segment coincides with the primary focus (the sun in a heliocentric system), while the other endpoint is the orbit apocenter. A spacecraft (or a celestial body) that tracks a heliocentric ERO experiences a rectilinear motion toward the sun with a purely radial velocity, that is, directed along the spacecraft-sun direction. The velocity magnitude is zero in correspondence of the orbit apocenter and takes its maximum value at the primary focus.

Apart from the theoretical mathematical interest of such orbits, the practical importance of a rectilinear trajectory toward the sun involves different possible scientific missions, such as those concerning the test of the equivalence principle, the analysis of the interstellar dust, or the study of the radial variation of the solar wind. In addition, an ERO could be useful for a detailed analysis of the sun's gravitational harmonics [2,3].

The main obstacle against the use of an ERO for scientific missions is mainly due to the substantial amount of propellant required to reach these orbits. To get a rough estimate, consider the minimum ΔV necessary to transfer a spacecraft from a heliocentric circular orbit of radius r_0 to a coplanar ERO with an aphelion radius $r_a > r_0$. Assuming a bi-impulsive maneuver and a Hohmann-like elliptic transfer orbit with semimajor axis $(r_0 + r_a)/2$, the total velocity variation necessary to complete the transfer is

$$\Delta V_{\min} = \sqrt{\frac{\mu_\odot}{r_0}}\left(\sqrt{2 - \frac{2r_0}{r_0 + r_a}} - 1\right) + \sqrt{\frac{\mu_\odot}{r_a}}\sqrt{2 - \frac{2r_a}{r_0 + r_a}} \quad (1)$$

For example, a transfer between a heliocentric terrestrial orbit ($r_0 = r_\oplus \triangleq 1$ AU) and an ecliptic ERO with aphelion radius equal to the Mars' mean heliocentric distance ($r_a \simeq 1.523$ AU) requires a total velocity variation of 24.43 km/s and a flight time of 0.7 years. Using the mission scenario proposed in [2], that is, an aphelion distance equal to the Jupiter's mean heliocentric distance ($r_a \simeq 5.2$ AU), a $\Delta V_{\min} \simeq 16$ km/s and a flight time equal to 2.73 years are obtained. The absolute minimum ΔV corresponds to $r_a \to \infty$, that is, when the ERO becomes a rectilinear parabola. In this (extreme) case a single-impulse orbit transfer in which the scientific probe attains the escape velocity with respect to the sun is obtained. Accordingly the transfer time becomes infinite, and Eq. (1) states that $\Delta V_{\min} = 12.33$ km/s when $r_0 = r_\oplus$.

A possible alternative to a high thrust propulsion system is offered by the employment of a solar sail. Indeed, the use of propellantless propulsion systems, such a solar sail, is known to be particularly attractive for those missions requiring large changes in orbital energy [4,5]. The idea of using a solar sail for reaching an ERO was originally proposed by Dandouras et al. [3]. However in that paper [3] the fulfillment of a rectilinear orbit is viewed as a secondary aspect of a scientific mission whose primary aim is to attain a heliostationary condition. Accordingly, in [3] the authors consider only very high-performance solar sails, whose characteristic acceleration is equal to (or greater than) the solar gravitational attraction. In this paper, instead, the problem is tackled within an optimal framework by looking for the minimum value of solar sail characteristic acceleration required to accomplish a mission toward an ERO. In this sense, for a given mission, a spacecraft performance increase corresponds to a characteristic acceleration decrease.

Mission Scenario

In the succeeding analysis it is intended that the term spacecraft comprises both the solar sail (the primary propulsion system) and the scientific probe. The baseline mission is ideally divided into two phases, a transfer phase (referred to as sailing mode), whose aim is to transfer the spacecraft from an initial parking orbit to the final rectilinear orbit, and an operative phase (referred to as scientific mode) in which the probe is used to perform the required scientific measurements.

Sailing Mode

At the initial time instant $t_0 = 0$ the spacecraft is placed on a circular parking orbit, with radius $r_0 = r_\oplus$, belonging to the ecliptic plane. This situation is representative of a vehicle at the end of an Earth escape phase with zero hyperbolic excess velocity with respect to the planet. Extensive numerical simulations have shown that the true eccentricity of the Earth's heliocentric orbit has a negligible effect on the mission performance. The spacecraft is transferred, within a given time interval t_1, to an ERO coplanar to the starting orbit. The coplanarity assumption not only simplifies substantially

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the mathematical analysis of the problem at hand, but also provides an estimate of the minimum solar sail performance necessary to attain the rectilinear orbit. In fact, any inclination variation between the circular and the rectilinear orbit would require an increase of the minimum solar sail characteristic acceleration a_c, which is the classical parameter used to characterize the solar sail performance [6]. In this paper a_c is defined as the maximum propulsive acceleration at $r = r_\oplus$.

The minimum value of a_c is found in an optimal framework, by minimizing, within a given time interval t_1, the characteristic acceleration necessary to set to zero the transverse component v of the spacecraft absolute velocity. During the problem solution, both the final distance from the sun $r_1 \triangleq r(t_1)$ and the spacecraft angular position θ along the ecliptic are left free, and constitute two outputs of the optimization process. The spacecraft radial velocity component $u_1 \triangleq u(t_1)$ at time t_1 is treated as a trajectory parameter. Its effect on the mission performance has been investigated by solving different optimal problems with the radial velocity ranging in the interval $u_1 \in [-10, 0]$ km/s. In particular, $u_1 = 0$ corresponds to a spacecraft that reaches the ERO aphelion at time t_1. In this case $r_1 = r_a = 2a$, where the semimajor axis a of the rectilinear orbit is

$$a = \frac{\mu_\odot r_1}{2\mu_\odot - r_1 u_1^2} \tag{2}$$

During the time interval in which the velocity magnitude is close to zero, the spacecraft experiences a near-heliostationary condition [7]. On the contrary, a velocity value $u_1 < 0$ implies that the orbital insertion is at a point between the two foci. According to Eq. (2), in this case $r_1 < 2a$.

The problem of minimizing a_c has been solved using an indirect approach, following the methodology described in [8]. The solar sail propulsive thrust has been simulated with both an ideal and an optical force model [6]. The equations of motion, the adjoint equations and the optimal control law are fully described in [9,10]. Finally, the boundary conditions for the orbital parameters that are left free, have been calculated by enforcing the transversality condition [11].

Figure 1 shows the minimum value of the characteristic acceleration a_c necessary to complete the transfer within a time interval $t_1 \in [1, 10]$ year using an ideal (that is, perfectly reflecting) flat solar sail assuming $u_1 \in [-10, 0]$ km/s. Figure 1, which has been obtained by solving the problem for 1000 different pairs (t_1, u_1), shows that the characteristic acceleration decreases as the flight time increases. The reason is that the minimization of a_c, for a given value of t_1, amounts to the problem of minimizing t_1 with a_c maintained fixed.

Some interesting conclusions can be drawn from Fig. 1. To reduce the computational effort, assume that the analysis is confined to missions with flight times on the order of 5 years, a value compatible with the operating life of a high-performance solar sail (as, for example, a solar sail required for escaping from the Solar System [12,13]). In this scenario, the characteristic acceleration is between 2.5 mm/s^2 and 3 mm/s^2. These numbers, albeit fairly high if compared with the current technology [14–16], correspond to less than one half of the values necessary for a heliostationary mission [3,7]. Moreover, Fig. 1 shows that the function $a_c = a_c(t_1)$ is not very sensitive to t_1 for flight times between 5 and 10 years. In fact, doubling the flight time (10 years), the decrease of characteristic acceleration is, independent of u_1, about 0.5 mm/s^2. Note, however, that these results do not take into account the degradation of the reflecting film material caused by a prolonged exposition to the interplanetary environment [17,18].

In the second place, Fig. 1 shows that the characteristic acceleration decreases by increasing $|u_1|$. Such a performance improvement is nearly independent of the flight time, in the sense that the derivative $\partial a_c/\partial u_1$ is nearly constant for different values of t_1. However, an increase of $|u_1|$ introduces two negative effects. On one side r_1 tends to rapidly decrease, see Fig. 2. For example, assuming $t_1 = 5$ year, a variation from $u_1 = 0$ to $u_1 = -10$ km/s implies a reduction of the final radius r_1 from 8 to 4 AU (−50%). The same behavior is confirmed also from Fig. 3, which shows the changes in the trajectory shape for different values of u_1, assuming that the solar sail is jettisoned at $t_1 = 5$ year. Moreover, upon combining the information from Figs. 1 and 2, it is clear that flight times less than 3 years are unsuitable, in the sense that they would require high values of characteristic acceleration ($a_c > 3$ mm/s^2) and would provide small values of final radius ($r_1 < 5$ AU). In particular, a large value of r_1 is important for the mission operative phase [2].

The second negative effect caused by a high value of $|u_1|$ is due to a decrease in the operating time interval of the scientific mode. In fact, a high value of the initial radial velocity toward the sun $|u_1|$, coupled with a small value of r_1, causes an early attainment of a small radial distance from the sun, with a consequent untimely destruction of the scientific probe. This specific problem will be further investigated in the following section with the aid of a tradeoff analysis involving a_c, r_1, and u_1.

Figure 2 shows the time variation of r_1 and $\Delta\theta$, the angle swept by the spacecraft during its transfer. When the initial anomaly (for example $\theta_0 \triangleq 0$) and the values of t_1 and u_1 are given, the pair $(r_1, \Delta\theta)$ univocally characterizes the rectilinear orbit on the ecliptic.

It is interesting to quantify the difference of performance $\Delta a_c/a_c$ and $\Delta r_1/r_1$, in terms of characteristic acceleration and distance from the sun, respectively, that are obtained when the specular reflecting

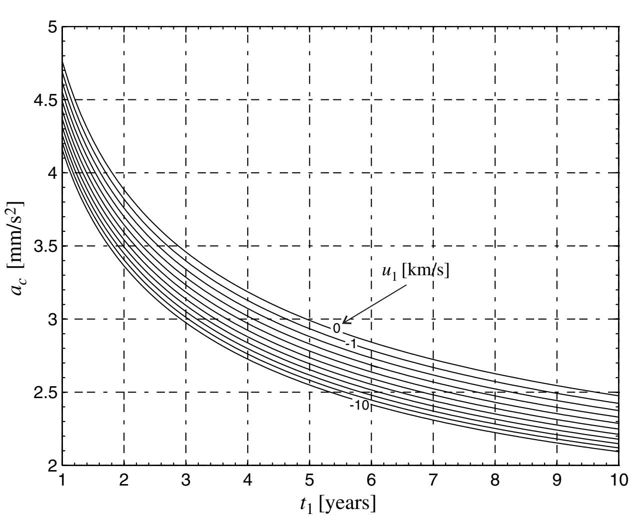

Fig. 1 Minimum characteristic acceleration a_c, for a coplanar transfer with an ideal solar sail, as a function of flight time $t_1 \in [1, 10]$ year and radial velocity $u_1 = (-10, -9, \ldots, 0)$ km/s.

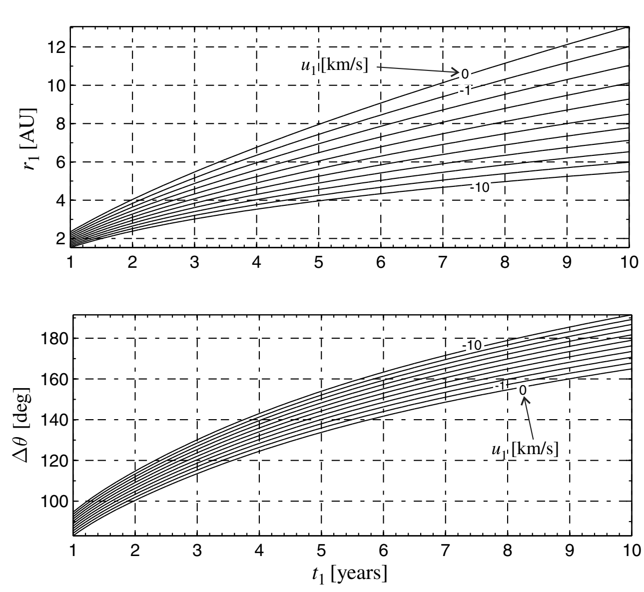

Fig. 2 Final radius r_1 and swept angle $\Delta\theta$, for a coplanar transfer with an ideal solar sail, as a function of the flight time $t_1 \in [1, 10]$ year and the radial velocity $u_1 = (-10, -9, \ldots, 0)$ km/s.

model is substituted by an optical force model [6]. Extensive simulations show that the percentage variation in characteristic acceleration is moderate, always less than 6.5%, with an absolute difference that does not exceed $\Delta a_c = 0.21\ \mathrm{mm/s^2}$. Therefore, for the sake of simplicity, only the ideal reflecting model will be used in all of the following simulations.

Scientific Mode

When the rectilinear orbit is reached, at time t_1, the scientific probe is released and begins its rectilinear motion toward the sun. According to Colombo et al. [2], the first phase of this mission segment should be used to calibrate the scientific instrumentation, the control systems and the communication with Earth. Therefore, it is important that the distance r_1 be reasonably long, and that the radial velocity magnitude $|u_1|$ does not exceed a few kilometers per second, to guarantee a sufficient time interval for the scientific measurements.

The scientific mission ends with the probe destruction caused by a close approach to the sun. Let $r_2 < r_1$ be the minimum tolerable distance from the sun and t_2 the time instant corresponding to the probe destruction. Using the Kepler's equation for a rectilinear motion it is possible to find a compact expression that relates the scientific phase time $t_2 - t_1$ to the minimum distance r_2. The result is:

$$t_2 - t_1 = \sqrt{\frac{a}{\mu_\odot}}[\sqrt{r_2(2a - r_2)} + a \arcsin(1 - r_2/a) - \sqrt{r_1(2a - r_1)} - a \arcsin(1 - r_1/a)] \tag{3}$$

where a is given by Eq. (2) as a function of the distance r_1 and the radial velocity u_1 at the beginning of the scientific mode. The final probe velocity $|u_2| > |u_1|$ at the end of the mission can be expressed as a function of the pair (r_1, u_1) with the aid of the energy equation:

$$u_2 = -\sqrt{u_1^2 + 2\mu_\odot\left(\frac{1}{r_2} - \frac{1}{r_1}\right)} \tag{4}$$

Equations (3) and (4) can be used to find the distance $r \in [r_2, r_1]$ and the velocity $u \in [u_2, u_1]$ of the probe at a generic time instant $t \in [t_1, t_2]$ during the rectilinear motion. To this end it is sufficient to symbolically substitute r_2 with r and t_2 with t in the two previous equations.

After the separation from the probe, the solar sail can be used for an extended mission. Potential scenarios are, for example, a flyby with a celestial body or a mission escape from the Solar System. The latter could be obtained by exploiting a trajectory with an orbital angular momentum reversal, as proposed by Vulpetti [19,20]. In fact the condition $v_1 = 0$, in which the transverse velocity component is set to zero, corresponds to setting the spacecraft angular momentum modulus h to zero. In other terms, the value of a_c obtained through the previous optimization process can be thought of as being equivalent to the minimum value of characteristic acceleration required to fulfil the condition $h = 0$ in a two-dimensional scenario. Alternatively, the solar sail could be inserted into a return trajectory toward the Earth. This concept would be useful, for example, for an in depth analysis of the degradation effect of the space environment on the solar sail film. In principle such a return trajectory would require a mission time smaller than $2t_1$, because the spacecraft mass is reduced with respect to the first mission phase. However, it should be noted that the sail film will also be degraded during the return trajectory. This degradation could, potentially, more than offset the mass reduction.

Preliminary Mission Design

Having fully characterized both the transfer phase and the scientific phase, it is now possible to analyze the whole mission. Firstly consider the dependence of the total mission time t_2 on the characteristic acceleration a_c and the radial velocity u_1. The function $t_2 = t_2(a_c, u_1)$ can be constructed by points using Eq. (3) for the scientific phase time, and the data taken from Fig. 1 for the transfer time. Figure 4 shows the contour curves (solid lines) that are obtained assuming a final solar distance $r_2 = 0.1$ AU [2], which corresponds to about 20 solar radii. The same figure also illustrates the contour curves (dashed lines) of the function $t_2 = t_2(a_c, r_1)$, extrapolated with the data taken from Fig. 2.

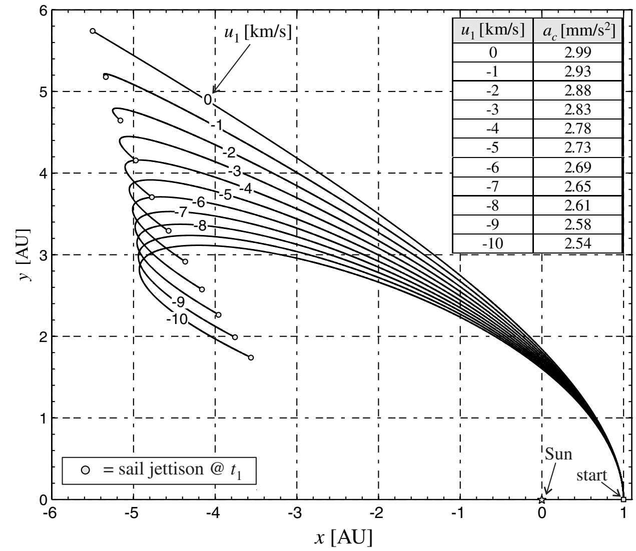

Fig. 3 Changes of trajectory shape for different values of $u_1 = (-10, -9, \ldots, 0)$ km/s (with $t_1 = 5$ year).

Missions with a total length of about 5 years are feasible with a characteristic acceleration less than $3.6\ \mathrm{mm/s^2}$. For example, assuming to start the scientific mode at a distance $r_1 \geq 5$ AU and enforcing a total flight time $t_2 \leq 5$ year, Fig. 4 states that such a mission requires a characteristic acceleration $a_c \in [3.16, 3.56]\ \mathrm{mm/s^2}$ and a radial velocity $u_1 \in [-3, 0]$ km/s. In this case the scientific phase length $t_2 - t_1$ is between 1.6 and 2.2 years, that is, between 32 and 44% of the total flight time.

A complete definition of the various mission parameters can be obtained with the aid of Fig. 5, which shows the contour curves of the scientific phase length $(t_2 - t_1)$ as a function of (t_1, u_1), (t_1, r_1), and (t_1, a_c). For example, for a mission total length $t_2 = 7$ year and a scientific phase $(t_2 - t_1) = 2$ year (28% of the total mission time) the minimum value of characteristic acceleration is about $2.78\ \mathrm{mm/s^2}$, with a radial velocity $u_1 = -4$ km/s and a distance $r_1 = 6$ AU.

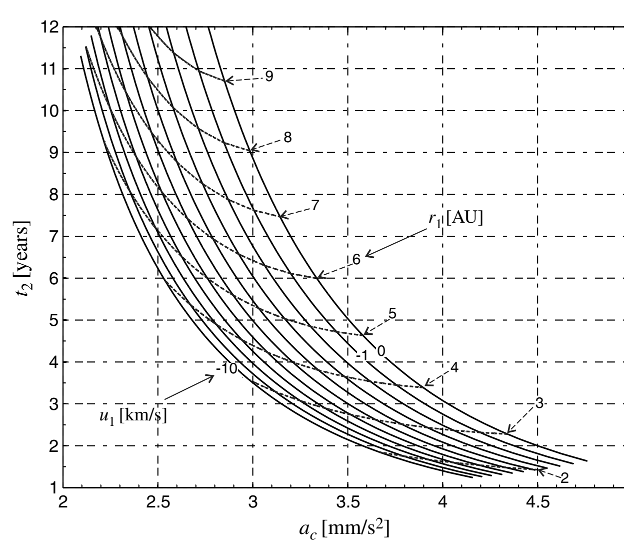

Fig. 4 Total mission time t_2 as a function of a_c, $u_1 = (-10, -9, \ldots, 0)$ km/s, and $r_1 = (3, 4, \ldots, 9)$ AU (with $r_2 = 0.1$ AU).

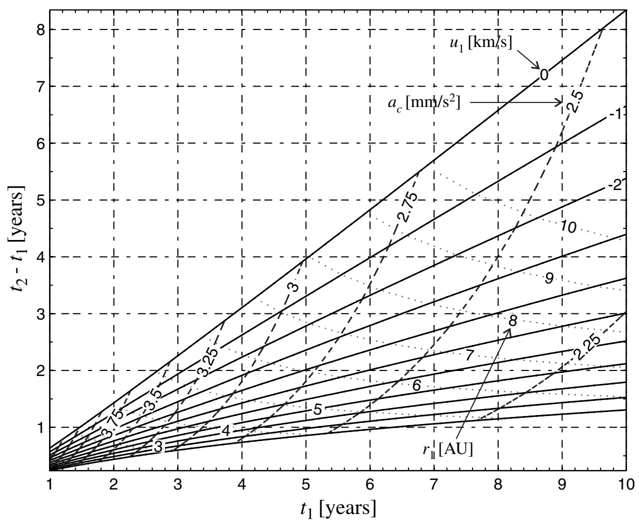

Fig. 5 Scientific phase length ($t_2 - t_1$) as a function of the spacecraft flight time t_1, radial velocity u_1, distance r_1, and characteristic acceleration a_c (with $r_2 = 0.1$ AU).

Conclusions

Missions toward elliptic rectilinear orbits have been studied in a parametric way. Different mission typologies have been investigated. The simulations results have been collected in graphs that can be used for obtaining a first estimate of the solar sail performance required to accomplish the mission, as well as for tradeoff analysis purposes. When the flight time is five years, the insertion into an elliptic rectilinear orbit requires a characteristic acceleration of about 3 $\mathrm{mm/s^2}$, which corresponds to a high-performance solar sail. Although such a value is beyond the current capabilities, the results motivate enhancing the current state of solar sail technology.

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